Key

Math 4

**4-8 Double Angle Formulas**

Name ____________________

Date______

*In this activity, you will be working towards the following learning goals:*

I can, without a calculator, use trigonometric identities such as angle addition/subtraction and double angle formulas, to express values of trigonometric functions in terms of rational numbers and radicals

***The Grand Finale . . .***

*The sum & difference formulas that we used in the previous investigations can be used to derive double angle formulas. In this investigation we will derive a total of six double angle formulas for sine, cosine & tangent.*

A. There is one formula for **sin (2*x*).**

To derive a formula for sin (2*x*), think of sin (2*x*) as **sin (*x* + *x*)**. Then use the sin (α + β) formula.

**sin (2*x*) = sin (*x* + *x*) =** $\sin x \cdot \cos x + \cos x \cdot \sin x$

$= \boxed{2\sin x \cdot \cos x}$

B. There are 3 formulas for **cos (2*x*).**

To derive a formula for cos (2*x*), use the same process you did in part A.

**cos (2*x*) = cos (*x* + *x*) =** $\cos x \cdot \cos x - \sin x \cdot \sin x$

$\sin^2 x + \cos^2 x = 1$

$= \boxed{\cos^2 x - \sin^2 x}$

Now use your result from above to write 2 more formulas for **cos (2*x*)** – one in terms of just sine & one in terms of just cosine.

**cos (2*x*) =** $\cos^2 x - (1 - \cos^2 x)$

$= \boxed{2\cos^2 x - 1}$

**cos (2*x*) =** $1 - \sin^2 x - \sin^2 x$

$= \boxed{1 - 2\sin^2 x}$

C. There are 2 formulas for **tan (2*x*).**

To get the first one, "stack up" sin (2*x*) & cos (2*x*).

**tan (2*x*) =** $\boxed{\dfrac{2\sin x\cos x}{\cos^2 x - \sin^2 x} = \dfrac{\sin 2x}{\cos 2x}}$

To derive the second one for tan (2*x*), use the same process you did in part A.

**tan (2*x*) = tan (*x* + *x*) =** $\dfrac{\tan x + \tan x}{1 - \tan x \cdot \tan x} = \boxed{\dfrac{2\tan x}{1 - \tan^2 x}}$

Transfer all six formulas you derived on the front of this page, below.
*Call me over to your group for verification of your formulas.*

$\cos 2x =$ $\cos^2 x - \sin^2 x$

$\sin 2x =$ $2 \sin x \cos x$

$\cos 2x =$ $2\cos^2 x - 1$

$\tan 2x =$ $\dfrac{2\sin x \cos x}{\cos^2 x - \sin^2 x} = \dfrac{\sin 2x}{\cos 2x}$

$\cos 2x =$ $1 - 2\sin^2 x$

$\tan 2x =$ $\dfrac{2\tan x}{1-\tan^2 x}$

**Check with me to make sure you are correct, then try examples 1 - 4.** ***No calculator!***

1. If $\cos x = \frac{4}{5}$, find $\cos 2x$.

NOT $x = \frac{4}{5}$

$\cos 2x = 2\left(\frac{4}{5}\right)^2 - 1$

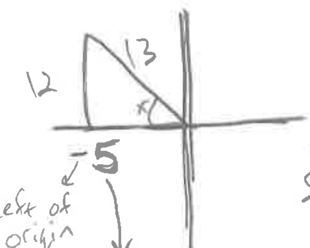
$= 2\left(\frac{16}{25}\right) - 1$

$= \frac{32}{25} - \frac{25}{25} = 1$

$= \boxed{\frac{7}{25}}$

2. If $\sin x = \frac{12}{13}$ and $\frac{\pi}{2} < x < \pi$, find $\sin 2x, \cos 2x$ and $\tan 2x$.

$2^{nd}$ Quad.




Left of origin

$a^2 + 12^2 = 13^2$

$a^2 = 25$

$a = 5$

$\sin x = \frac{12}{13}$

$\cos x = -\frac{5}{13}$

$\tan x = -\frac{12}{5}$

$\sin 2x = 2 \sin x \cdot \cos x$

$= 2\left(\frac{12}{13}\right)\left(-\frac{5}{13}\right)$

$= \boxed{\frac{-120}{169}}$

$\cos 2x = 2\cos^2 x - 1$

$= 2\left(-\frac{5}{13}\right)^2 - 1$

$= 2\left(\frac{25}{169}\right) - 1$

$= \frac{50}{169} - \frac{169}{169}$

$= \boxed{\frac{-119}{169}}$

$\tan 2x = \dfrac{2\left(-\frac{12}{5}\right)}{1-\left(-\frac{12}{5}\right)^2}$

$\dfrac{\frac{-24}{5}}{}$

$= \dfrac{}{\frac{25}{25} - \frac{144}{25}}$

$= \dfrac{-24}{\cancel{5}} \cdot \dfrac{\cancel{25}^{5}}{-119}$

$\dfrac{-119}{25}$

$= \boxed{\frac{120}{119}}$

In what quadrant does $2x$ lie? How do you know?

$3^{rd}$! Both $\sin 2x$ + $\cos 2x$ are negative.

3. Solve for primary values: $2\cos x + \sin 2x = 0$

$2\cos x + 2\sin x \cos x = 0$

$2\cos x\,(1 + \sin x) = 0$

$2\cos x = 0$

$\cos x = 0$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$

$1 + \sin x = 0$

$\sin x = -1$

$x = \frac{3\pi}{2}$

4. Verify: $\cos^4 x - \sin^4 x = \cos 2x$

$$\underbrace{(\cos^2 x + \sin^2 x)}(\underbrace{\cos^2 x - \sin^2 x})$$

$$= 1 \cdot \cos 2x$$

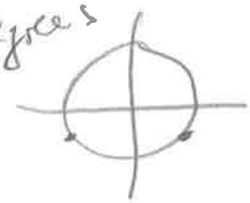
$$= \boxed{\cos 2x}$$

Graph first! Xmin: 0, Xmax: $2\pi$ → See 4 x-int.

Ymin: −1, Ymax: 3

Answer in degrees

☆ 5. $2\sin(2x) + 1 = 0$



$u = 2x$

$2\sin u = -1$

$\sin u = -\frac{1}{2}$

$u = 210°, 330°$

The 2x makes the period go from $2\pi$ to $\pi$ (180°)

→ Primary values are any solutions from $0 - 2\pi$ or $0 - 360°$

$2x = 210°$ $\quad$ $2x = 330°$

$\boxed{x = 105°}$ $\quad$ $\boxed{x = 165°}$

$x = 65 + 180 = \boxed{285°}$ $\quad$ $x = 165 + 180 = \boxed{345°}$

☆ 6. $2\sin^2(2x) - 5\sin(2x) = 3$

$u = 2x$

$2\sin^2 u - 5\sin u - 3 = 0$

$(2\sin u + 1)(\sin u - 3) = 0$

Period $= \pi$

Still a primary value!

$\sin u = \frac{-1}{2}$ $\quad$ $\sin u = 3$

$\boxed{\text{No solution}}$

$u = \frac{7\pi}{6}, \frac{11\pi}{6}$

$2x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\boxed{x = \frac{7\pi}{12}, \frac{11\pi}{12}}$ & $\frac{7\pi}{12} + \pi \left(\frac{12\pi}{12}\right) = \boxed{\frac{19\pi}{12}}$ + $\frac{11\pi}{12} + \frac{12\pi}{12} = \boxed{\frac{23\pi}{12}}$